Auto Encoders

What PCA wished it was

Introduction to Machine Learning, Spring 2026

Recall: Motivation for PCA

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Lower dimension $\rightarrow$ computational efficiency, lower redundancy (list goes on)

What can go wrong?

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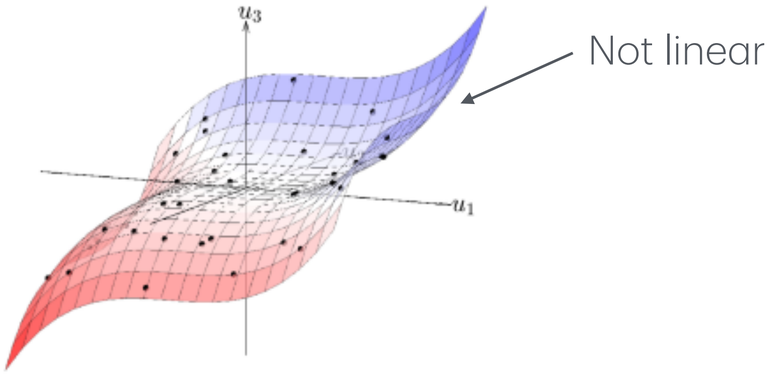
Assumes that data lies on a smaller dimensional *linear* manifold



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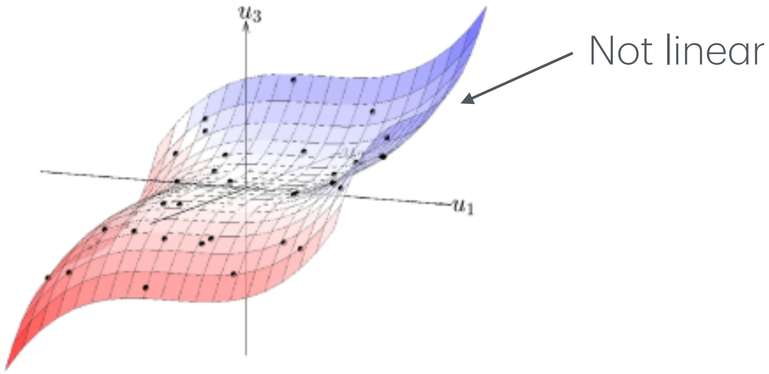

It is lossy - the lower dimensional representations cannot be used to get the original data back.

Can we *learn*?

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We need a clever way to convert this unsupervised learning task into a supervised learning task

A random problem

A random problem

Let's consider a completely different problem

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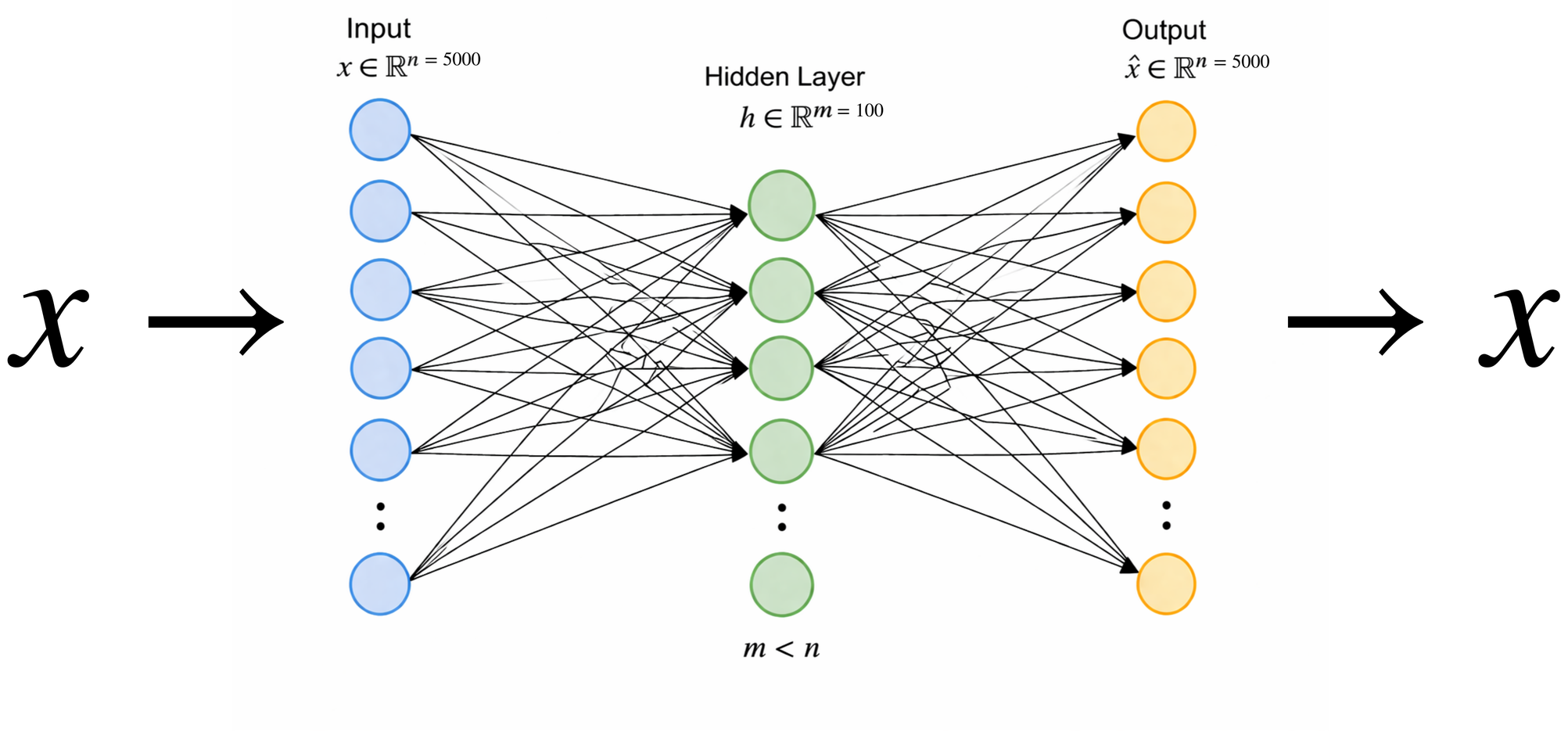
Suppose I train a network, such that given x, it outputs.. x

A random problem



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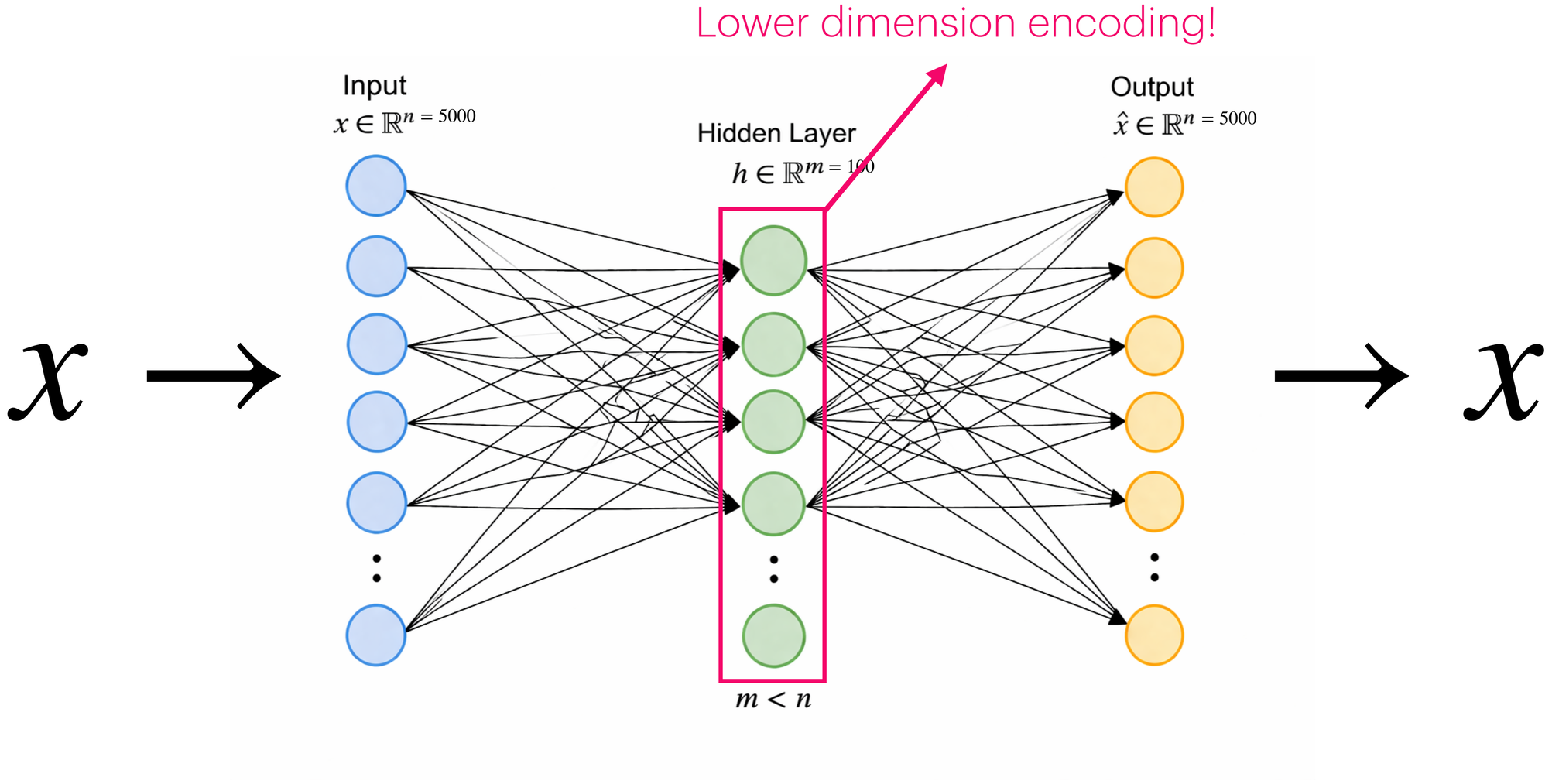

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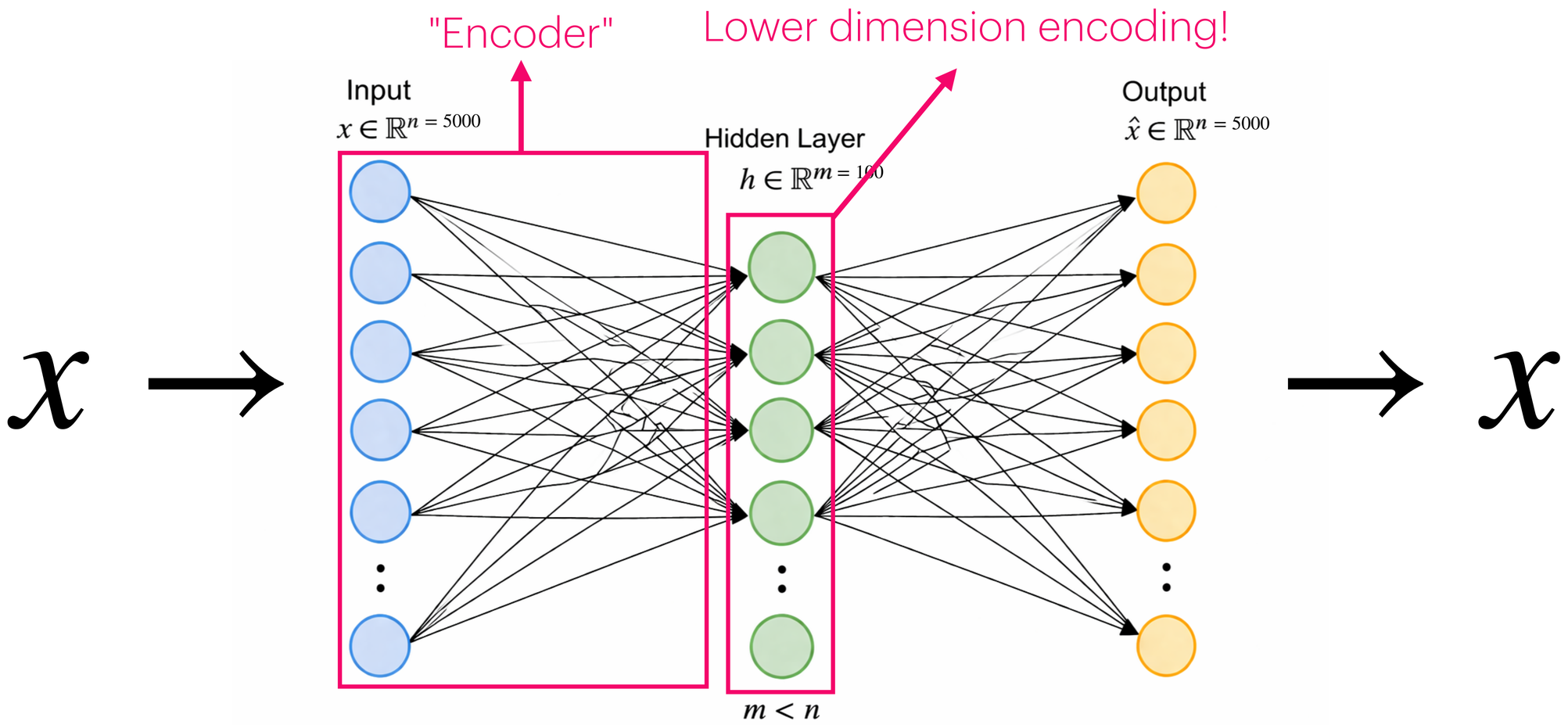

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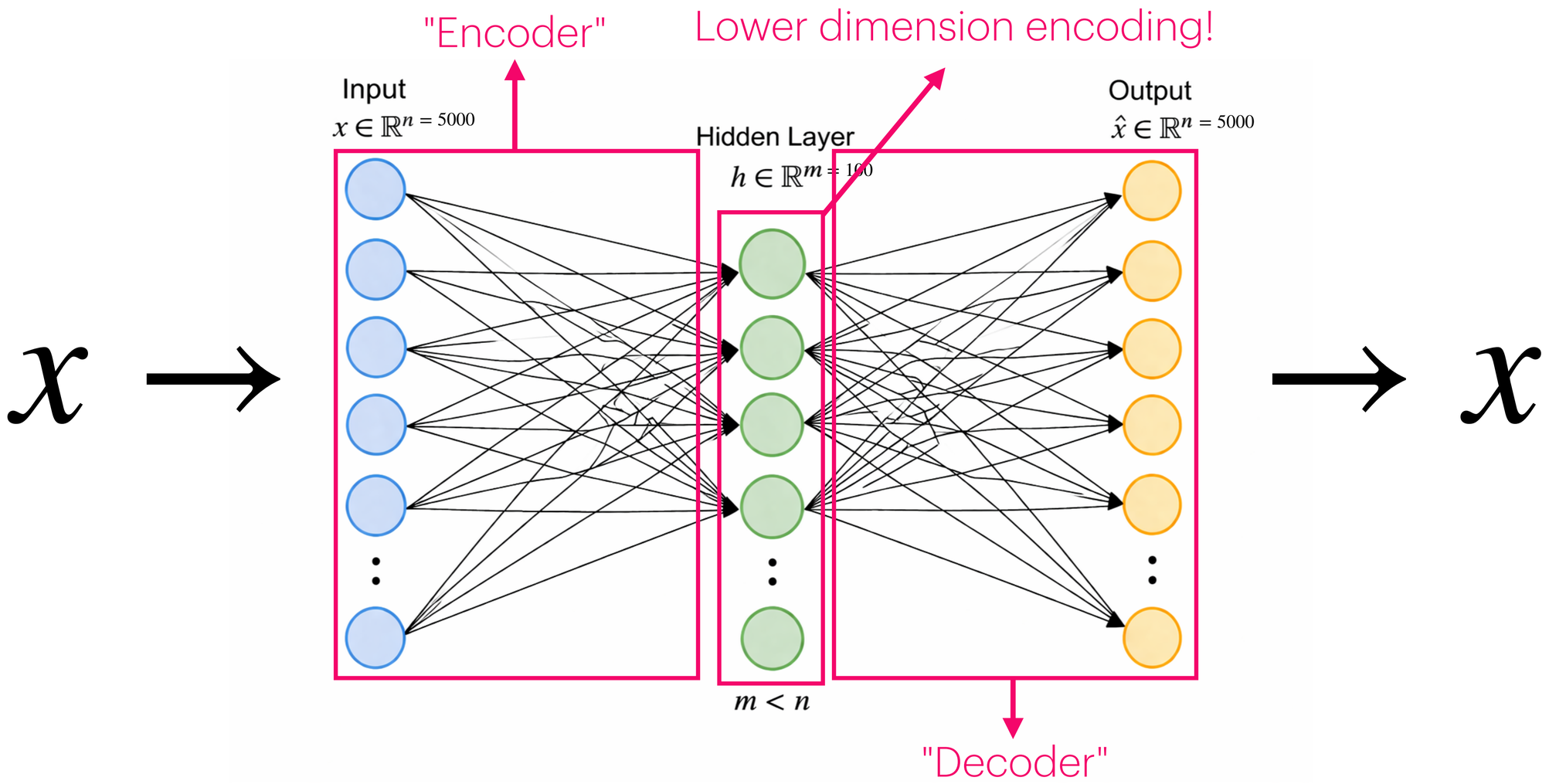

Simplifying the problem

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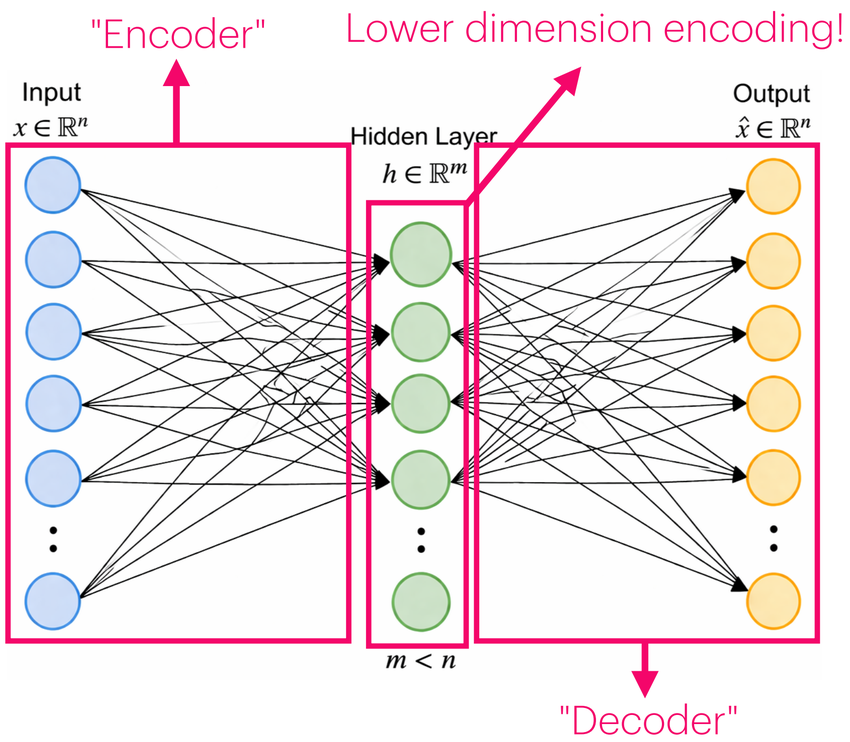

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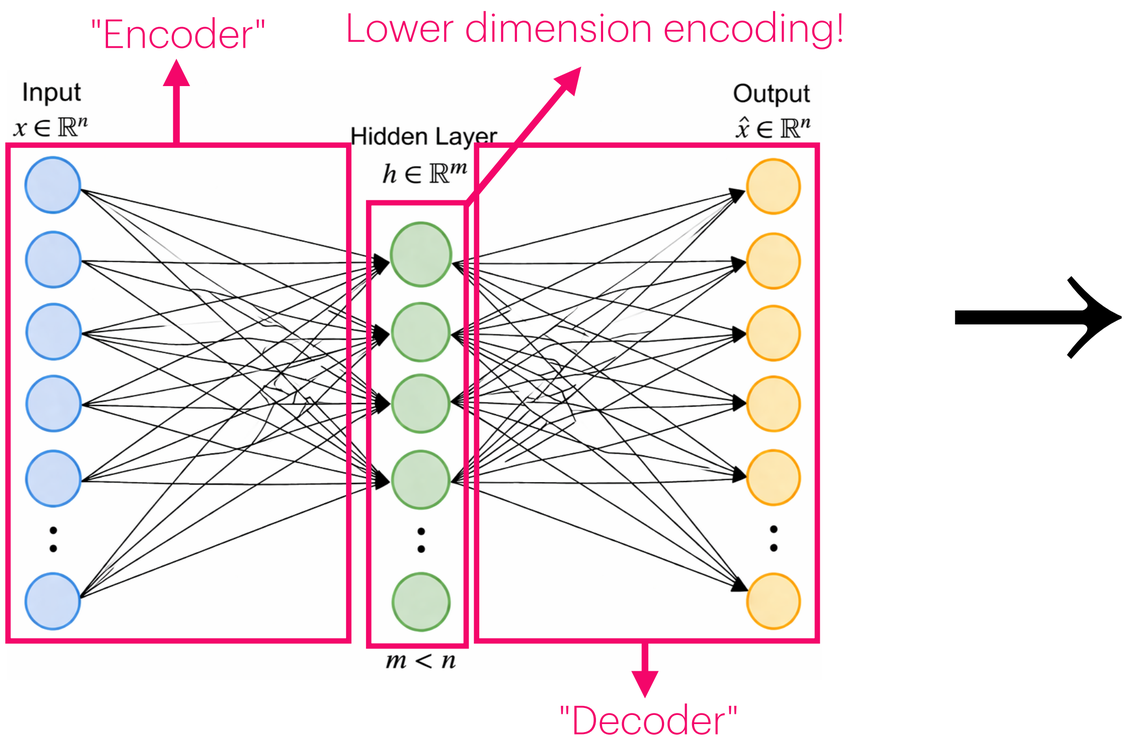

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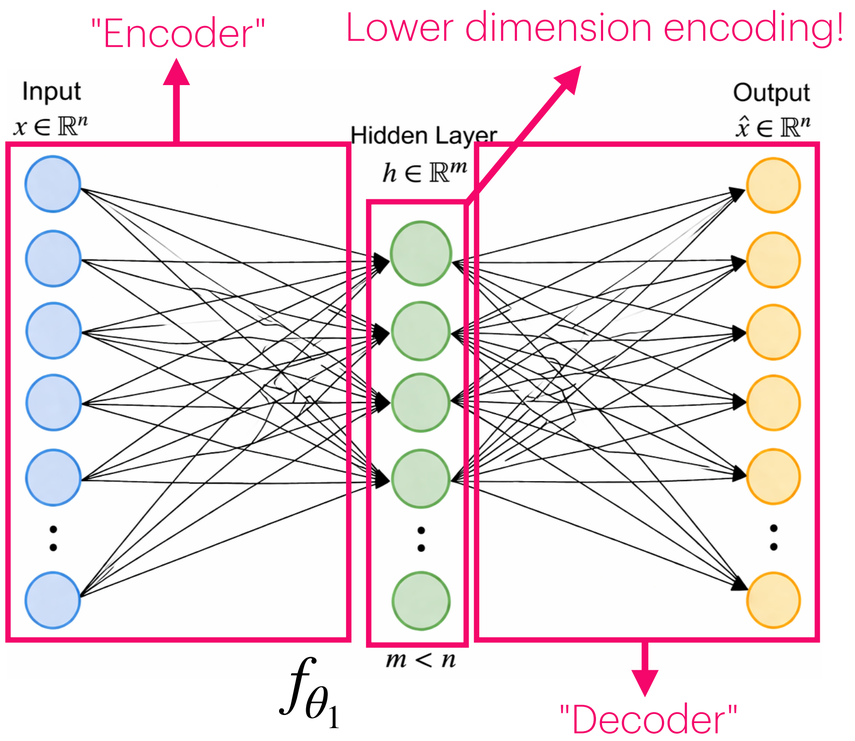

$f_{\theta_1} : \mathbb{R}^n \to \mathbb{R}^m$ – the encoder

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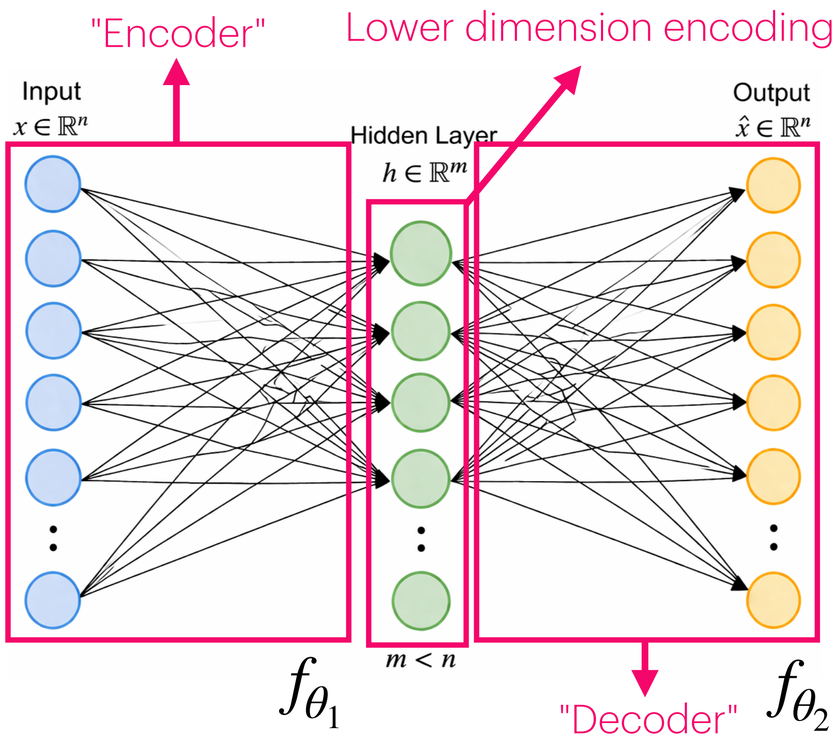

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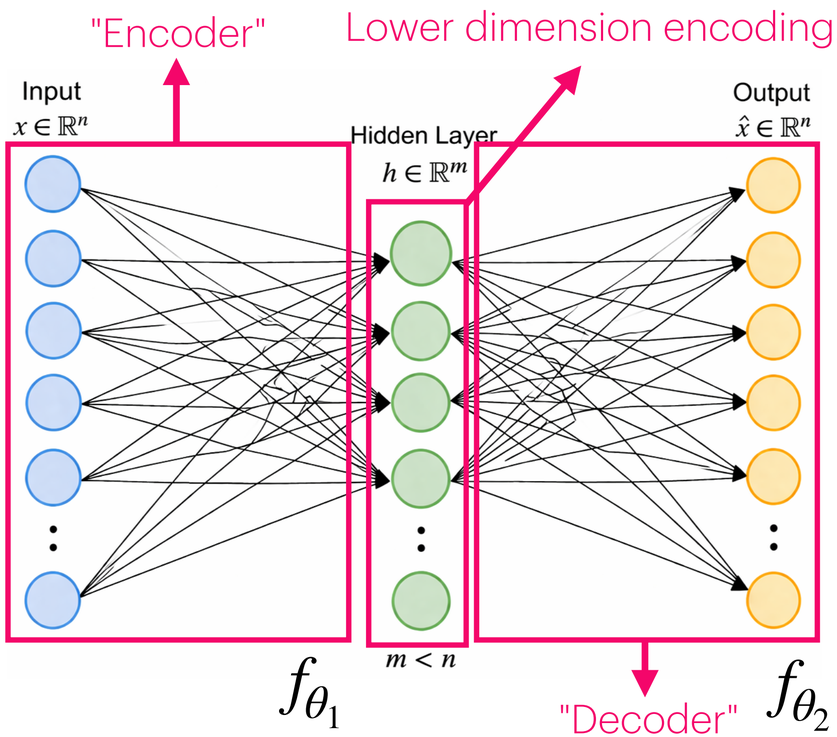

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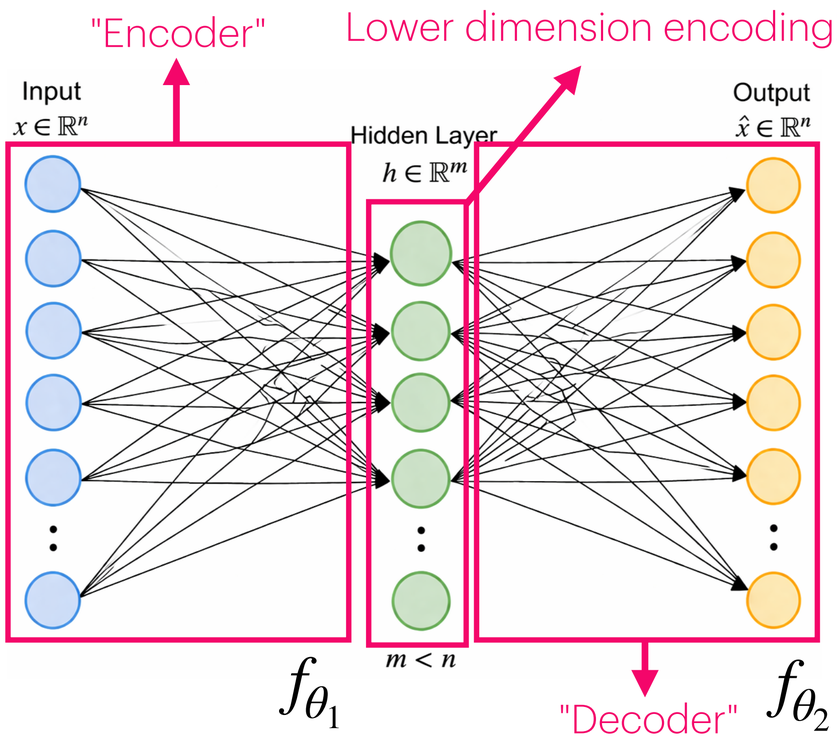

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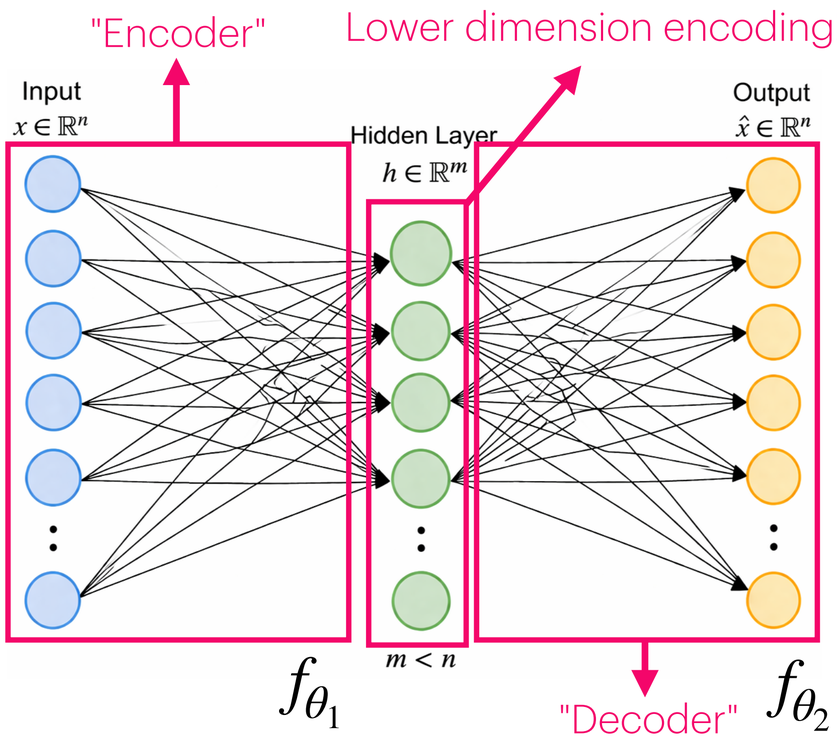

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The architecture

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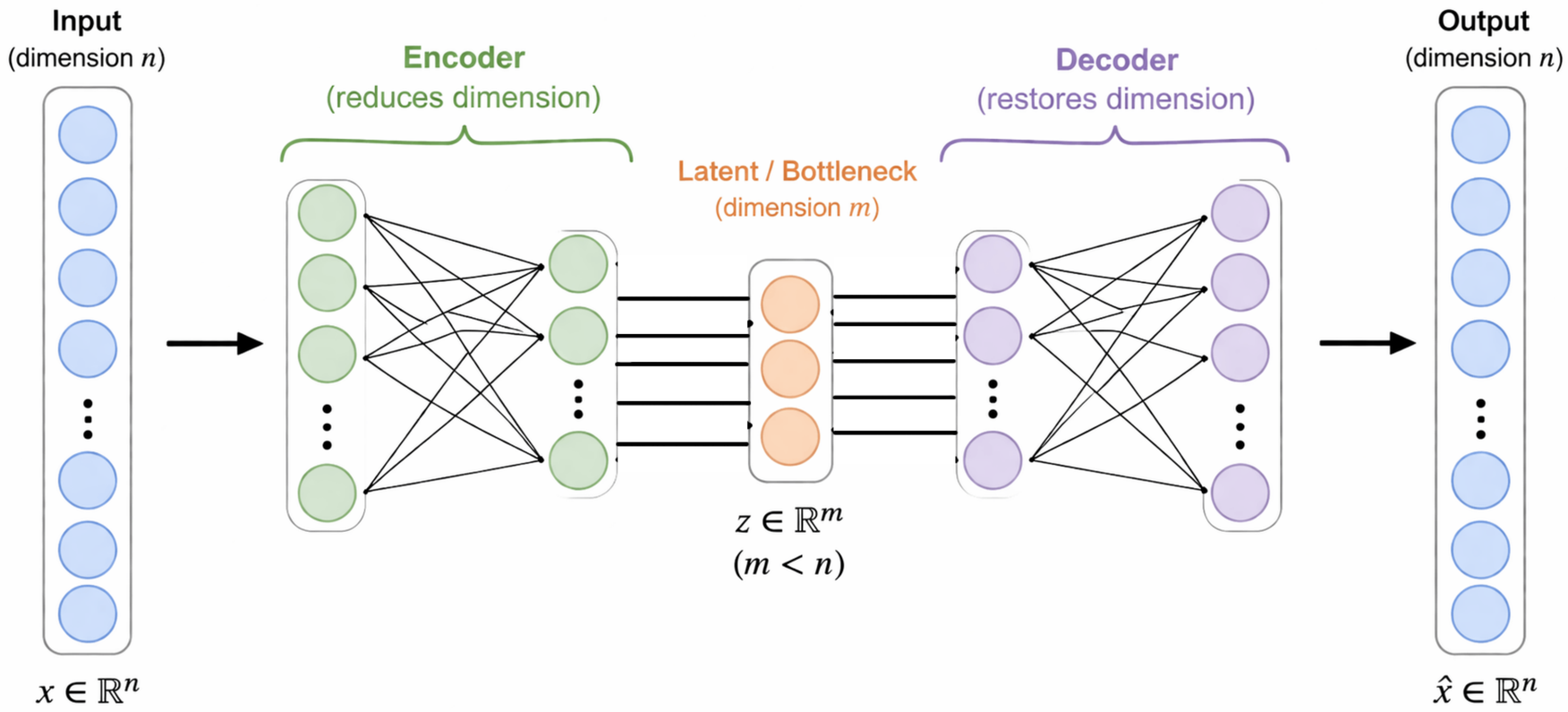

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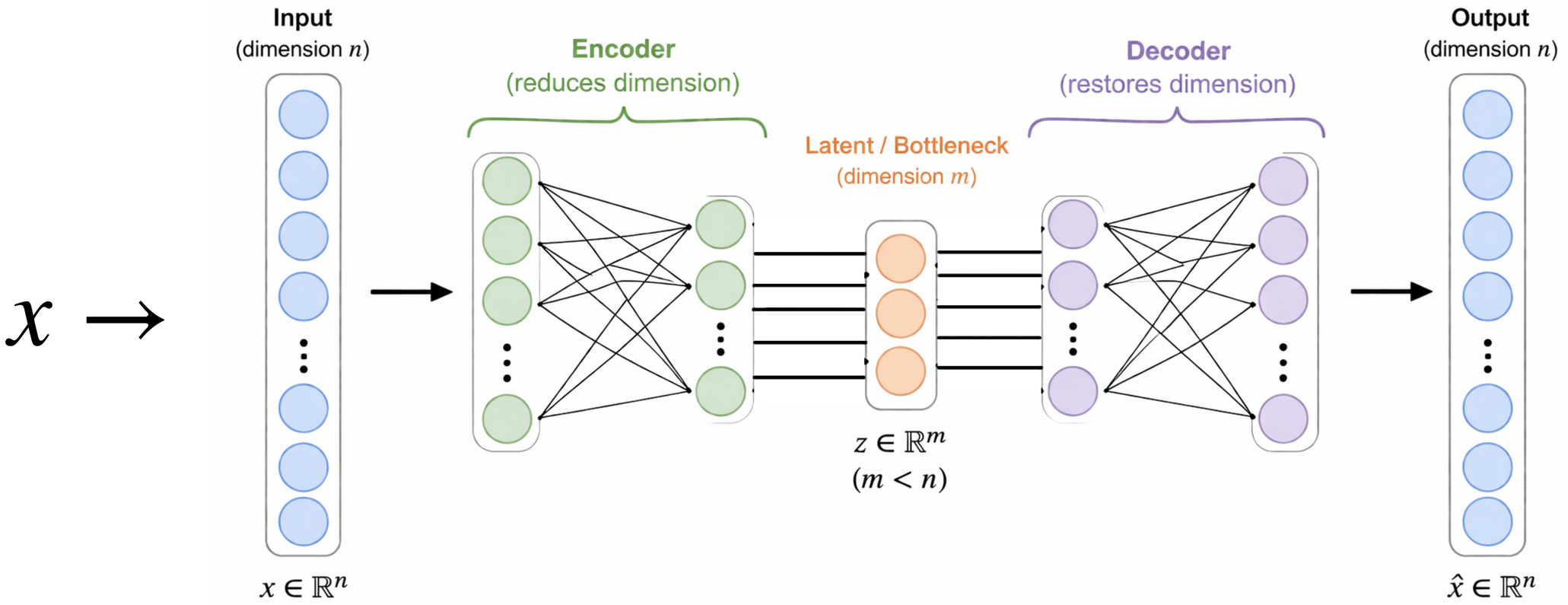

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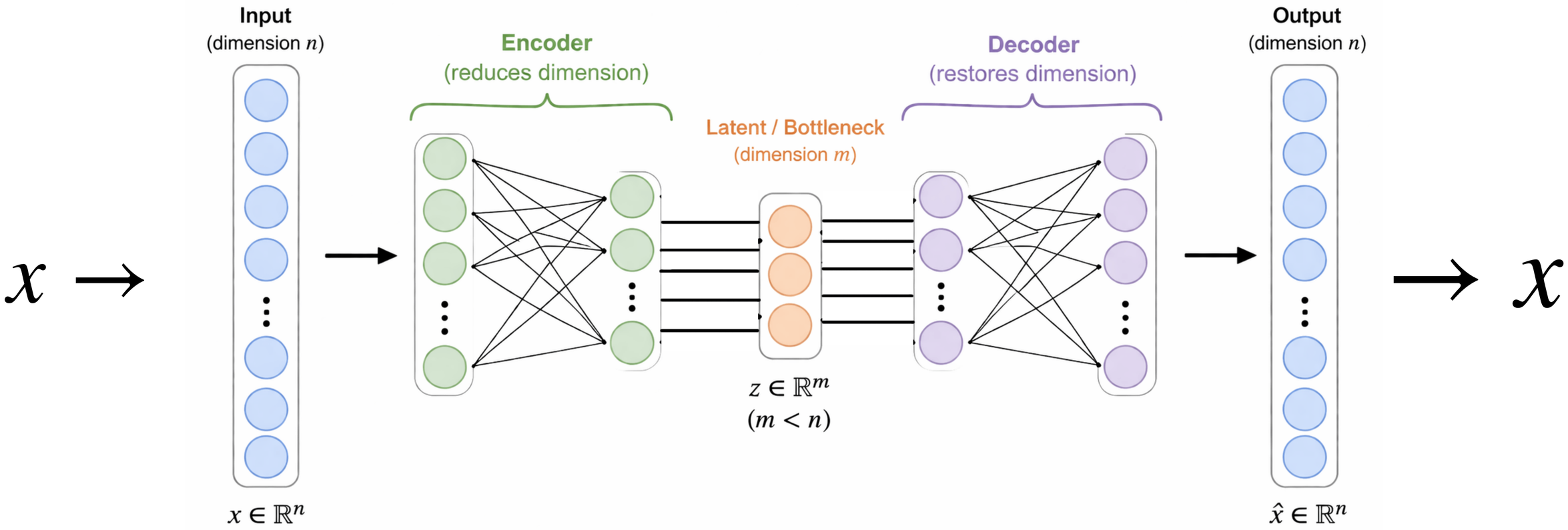

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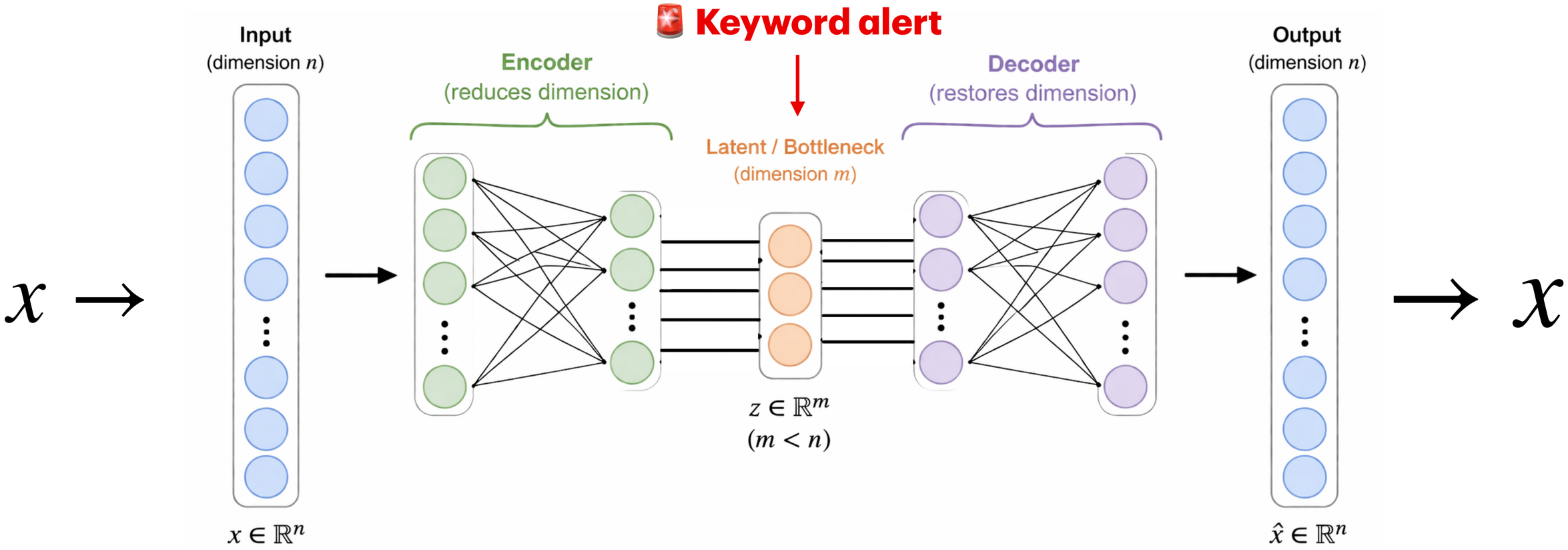

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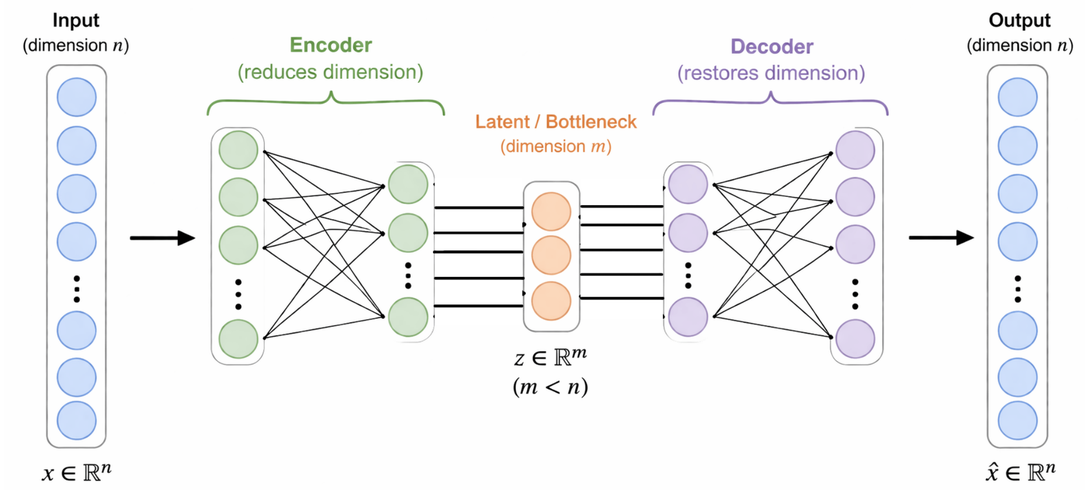

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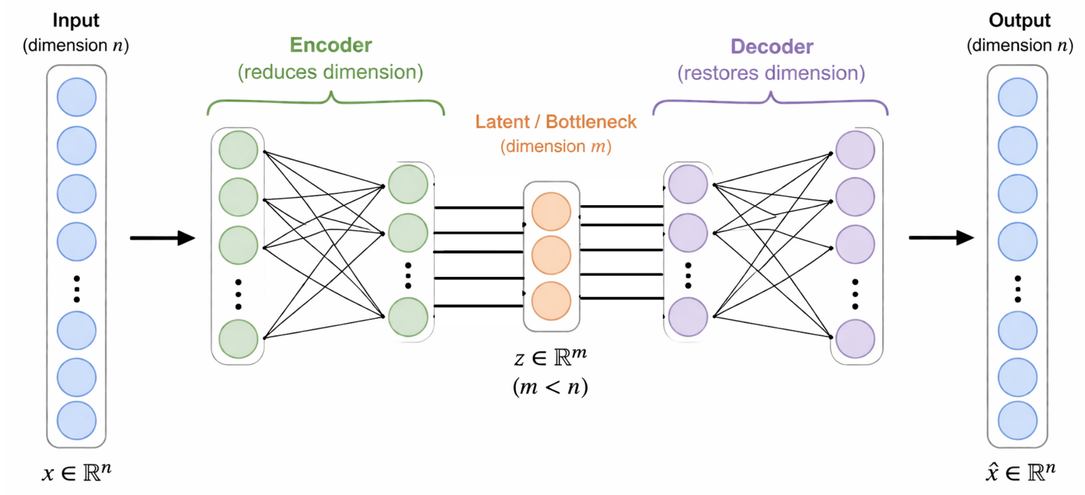
Hyperparameters?



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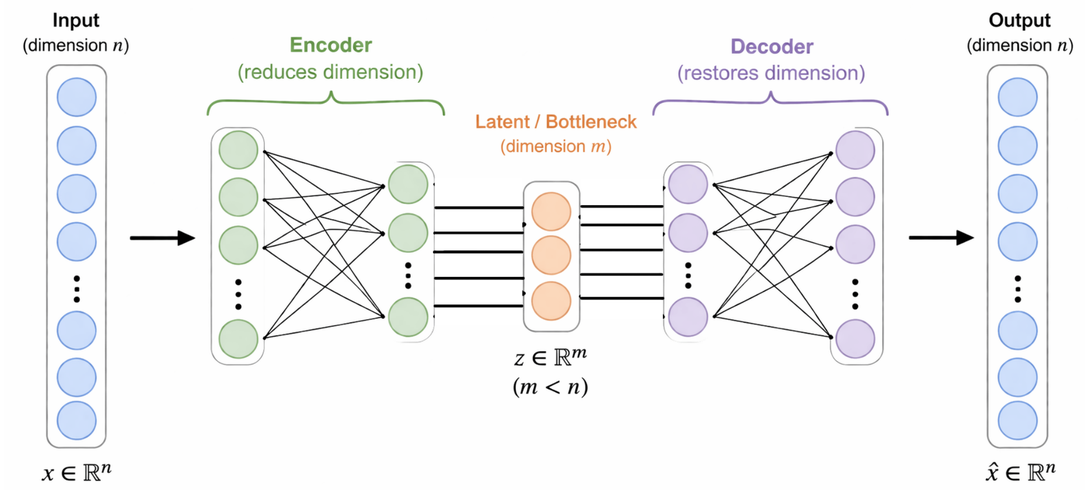

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Compression quality?

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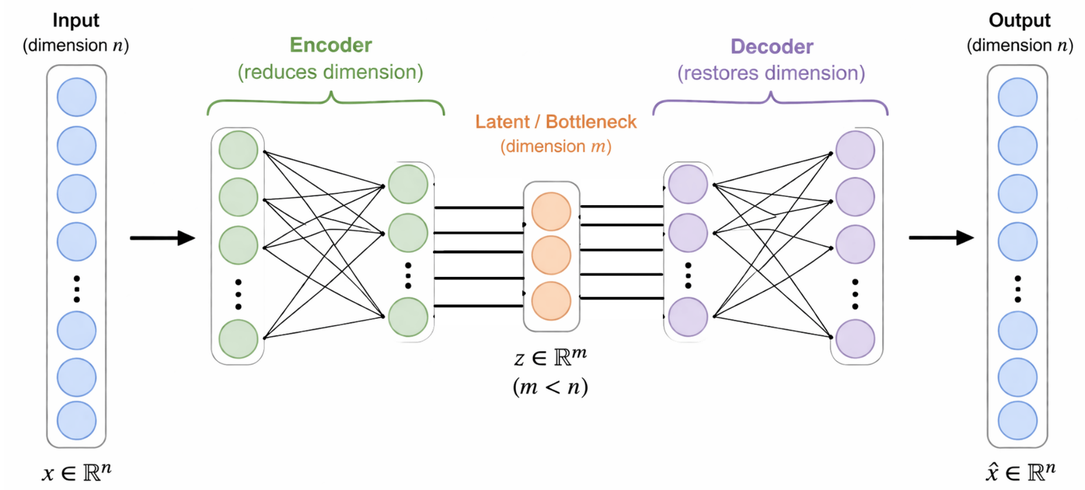

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Expressiveness of neural network. Size of latent dimension. Training duration

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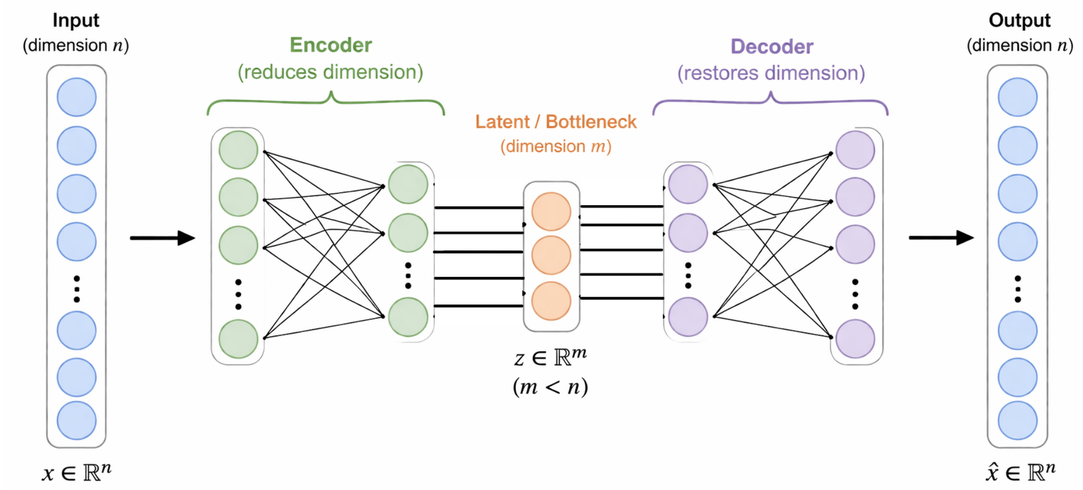

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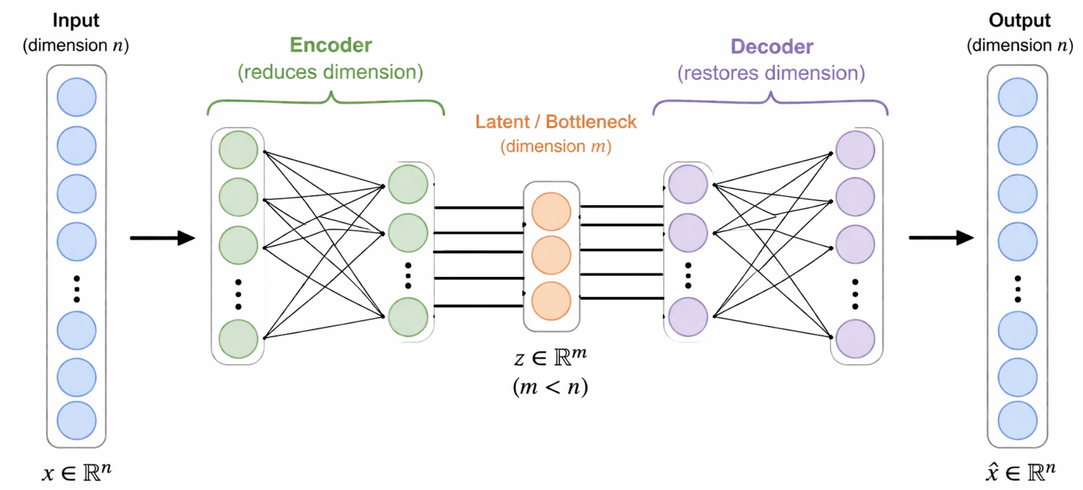

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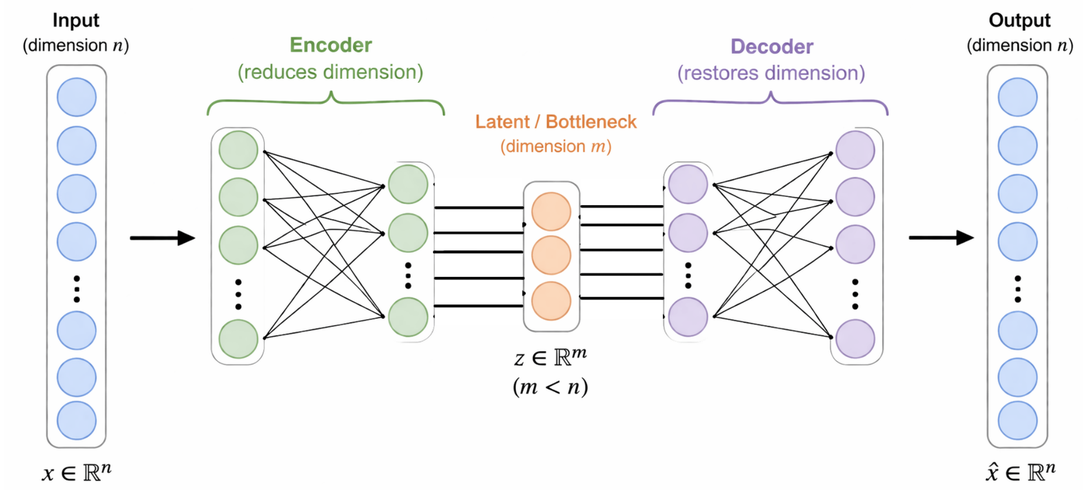

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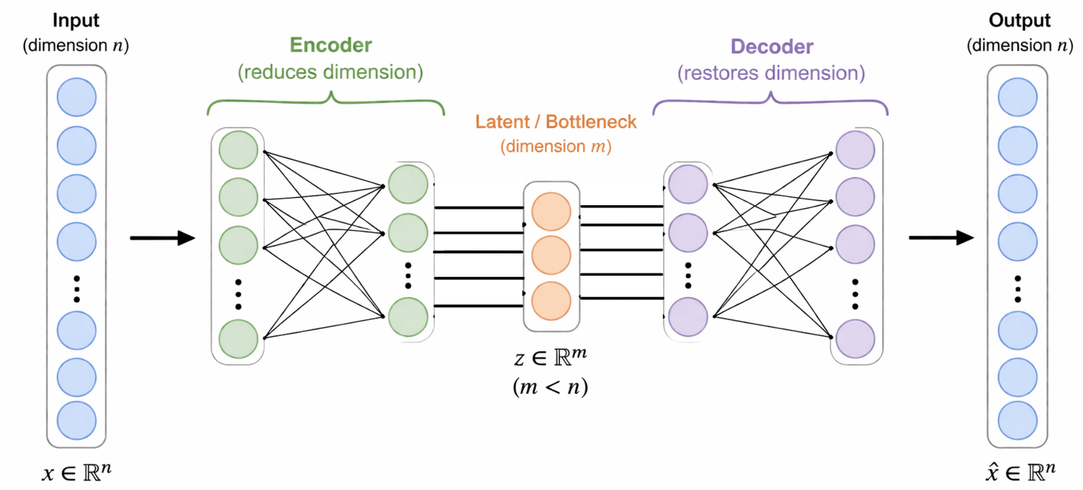

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Architecture and size of Network

Note on neural networks

We have seen neural networks as compositions of smaller systems:

- Layers (weights + biases)
- Residual (skip) connections
- Convolution Layers

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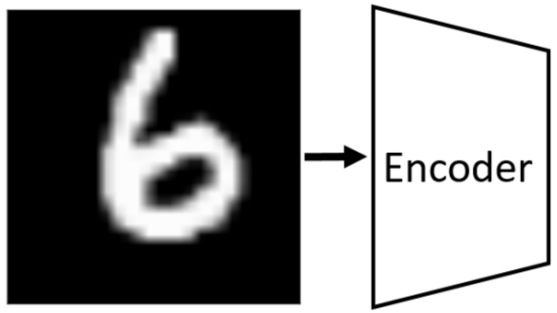
This shows a generalisation:

Neural networks can be built with neural networks as building blocks!

A concrete example

Original
input

A concrete example



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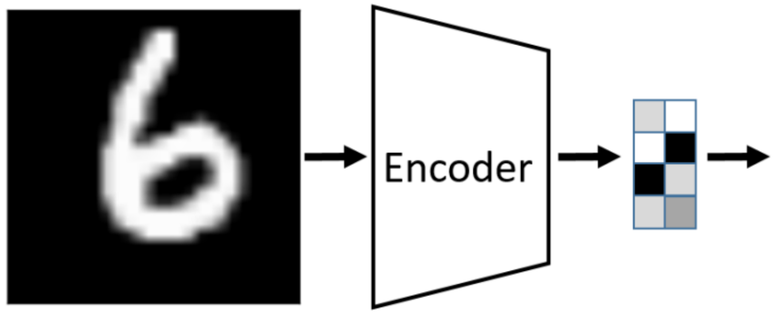
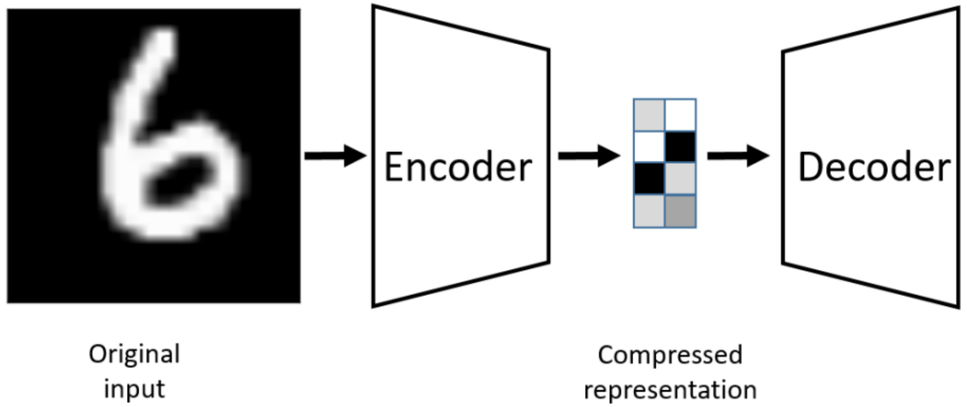


Original input

Compressed representation

A concrete example



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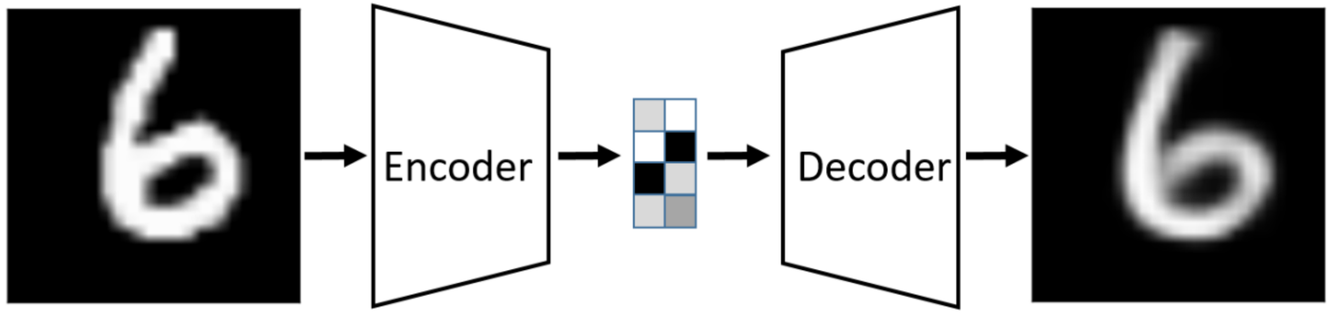

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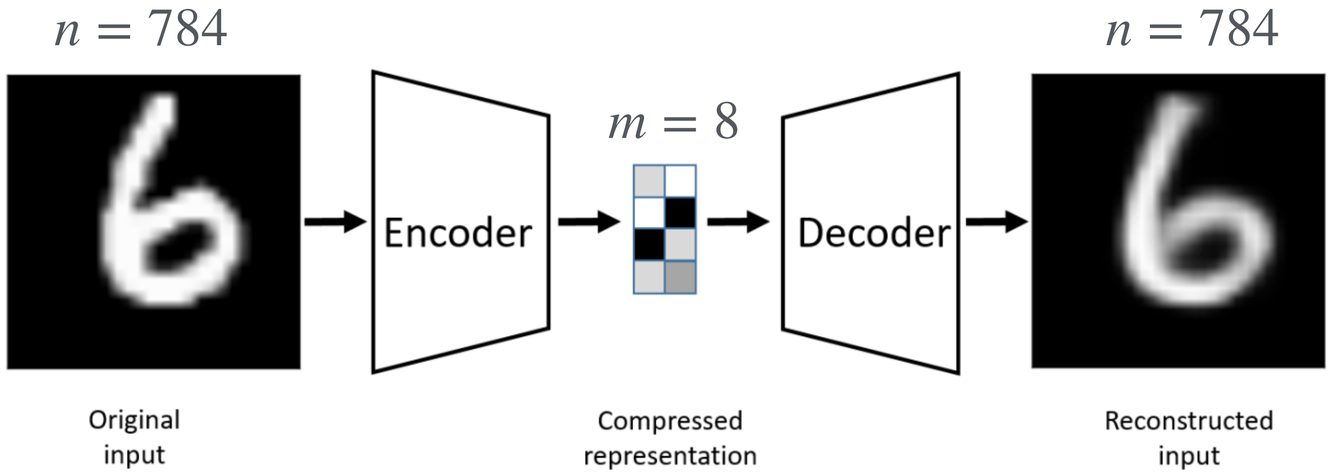

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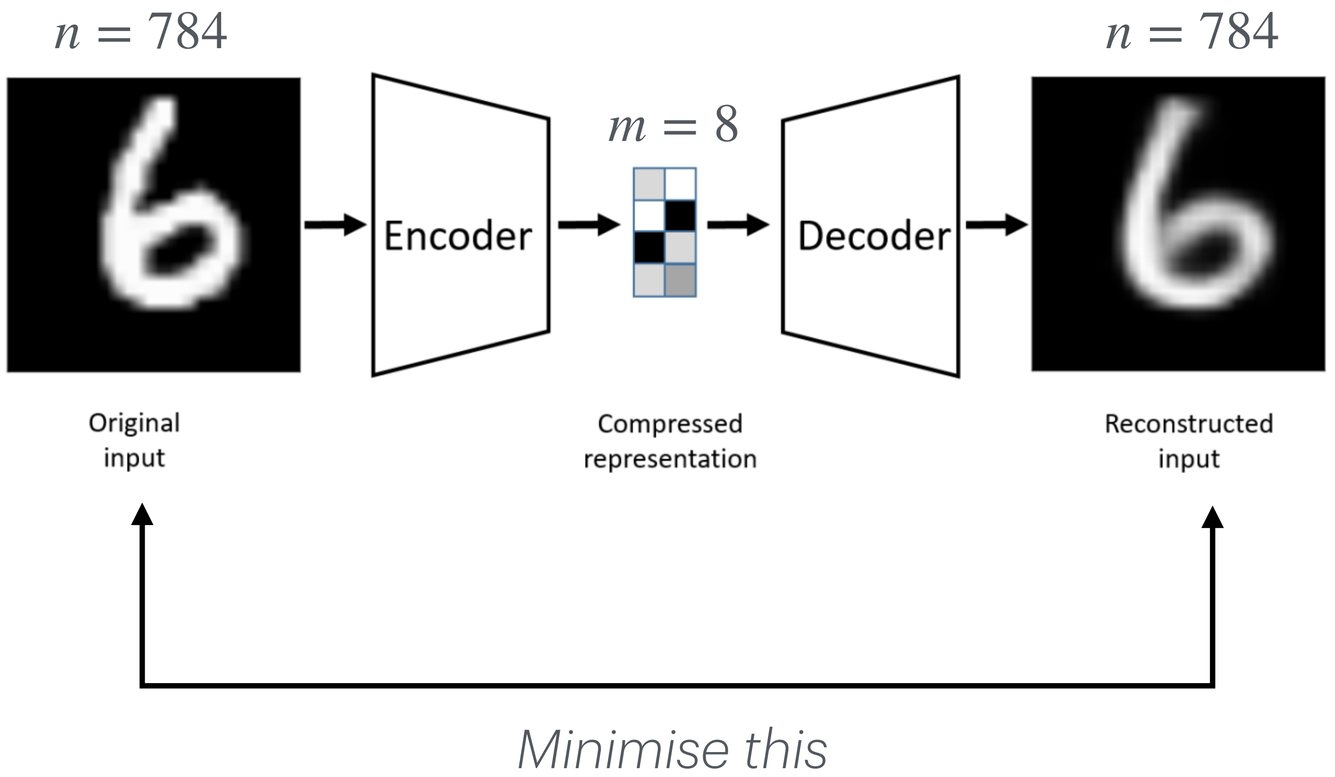

The optimisation problem

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$$\hat{L}(\theta_1, \theta_2) = \frac{1}{n} \sum_{i=1}^{n} \left\| f_{\theta_2}\left(f_{\theta_1}(x_i)\right) - x_i \right\|_2^2$$

The Algorithm

Given dataset X, encoder f_{θ_e}, decoder g_{θ_d}

for each epoch:

for each mini-batch $B \subset X$:

$$z = f_{\theta_e}(x)$$

$$\hat{x} = g_{\theta_d}(z)$$

$$\mathscr{L} = \|x - \hat{x}\|^2$$

$$\theta_e \leftarrow \theta_e - \eta \nabla_{\theta_e} \mathscr{L}$$

$$\theta_d \leftarrow \theta_d - \eta \nabla_{\theta_d} \mathscr{L}$$

Neural
Networks

AutoEncoder
with Encoder-Decoder
architecture

Variational Auto Encoders

The whole secret lies in confusing the enemy, so that he cannot fathom our real intent.
- Sun Tzu, “The Art of War”

Introduction to Machine Learning, Spring 2026

Deeper look

Deeper look

What does the the compression actually do?

Deeper look

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Autoencoders tend to have discrete latent spaces, where each input is mapped to some arbitrary point

So what?

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If we take a random point in the latent space and decode it, we get garbage

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The latent space does not capture the semantic meaning well

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The latent space does not capture the semantic meaning well

Can we make it a nicer space, maybe continuous?

Hunt for a better space

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The encoder predicts a *fixed* output, causing this memorisation

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Can we *vary* the output of the encoder?

Hunt for a better space

The encoder predicts a *fixed* output, causing this memorisation

Can we *vary* the output of the encoder?

Instead of encoding to fixed point, encode using some *randomness*

Randomness

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Randomness should be small enough for the decoder to reconstruct, but large enough for the encoder to not memorise

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But how?

Randomness

Randomness should be small enough for the decoder to reconstruct, but large enough for the encoder to not memorise

But how?

Just learn it

Learning the randomness to learn…

Learning the randomness to learn...

The randomness is parameterised by a Gaussian

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The encoder takes a data point and outputs the mean μ and the log variance $\log \sigma^2$ of the gaussian

Learning the randomness to learn...

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The latent is now a sample from this distribution

An example

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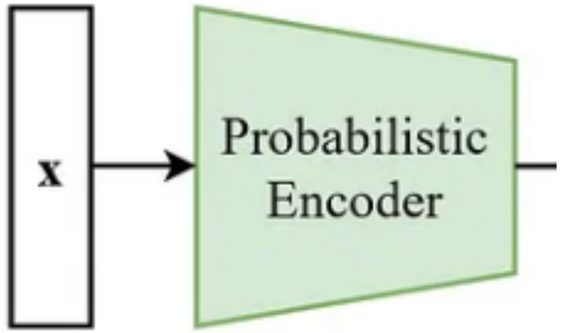

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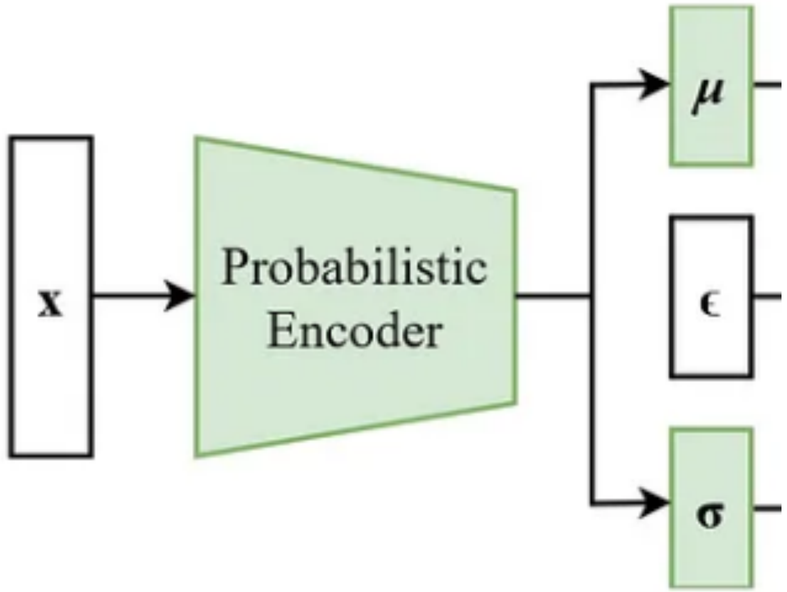

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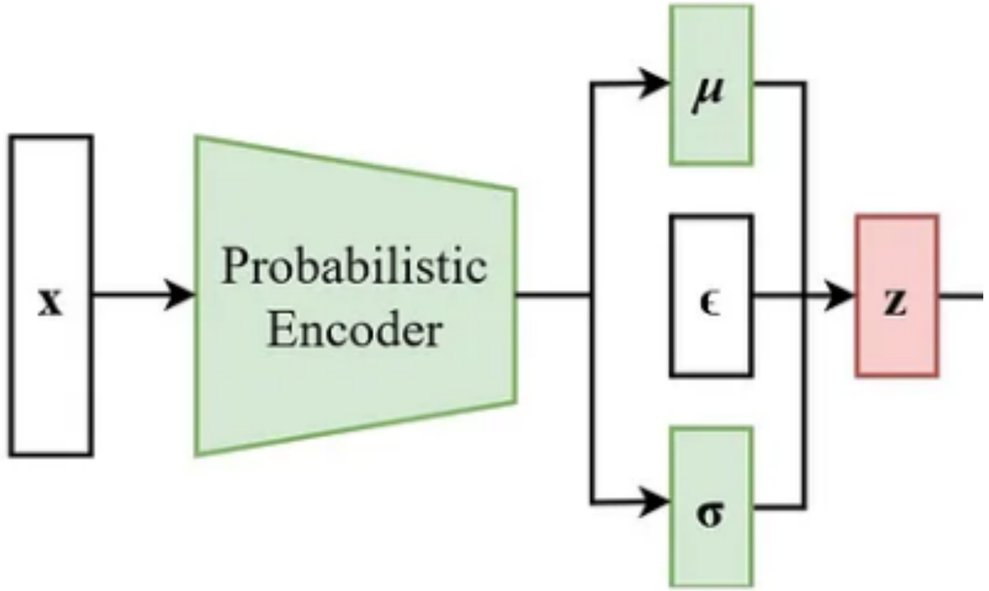

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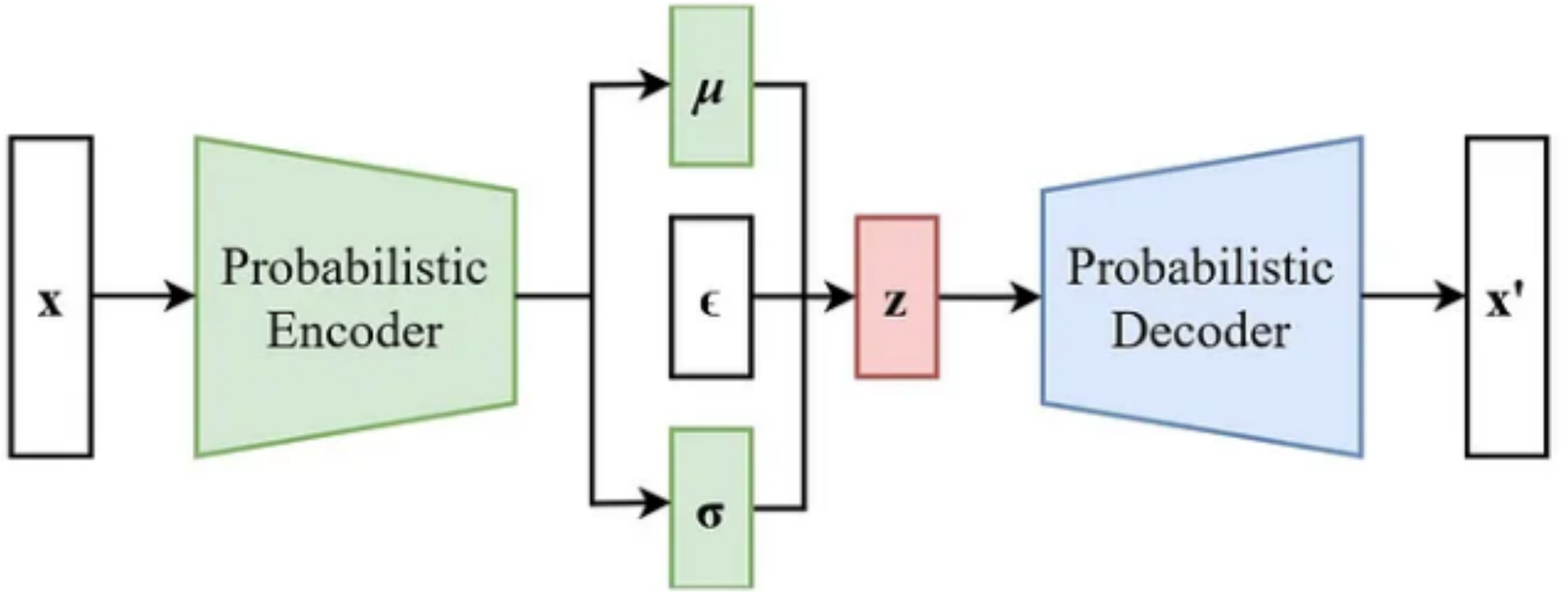

What can do wrong?

What can do wrong?

Model always sets the variance to 0, becomes an auto encoder

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Model always sets the variance to 0, becomes an auto encoder

Can we penalise the model for being too “discrete”?

The penalty

The penalty

Force the encoder to produce outputs close to a standard Gaussian by adding a KL loss

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Force the encoder to produce outputs close to a standard Gaussian by adding a KL loss

$$\mathrm{KL}\left(p||q\right) = \int p(x) \log \frac{p(x)}{q(x)} dx$$

Measures how close two distributions are

The closed form

$$p = \mathcal{N}(\mu_p, \sigma_p^2 I), \quad q = \mathcal{N}(\mu_q, \sigma_q^2 I)$$

The closed form

$$\mathrm{KL}\left(p \| q\right) = \frac{1}{2}\left[d\frac{\sigma_p^2}{\sigma_q^2} + \frac{\|\mu_p - \mu_q\|^2}{\sigma_q^2} - d + d\log\left(\frac{\sigma_q^2}{\sigma_p^2}\right)\right]$$

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Finally...

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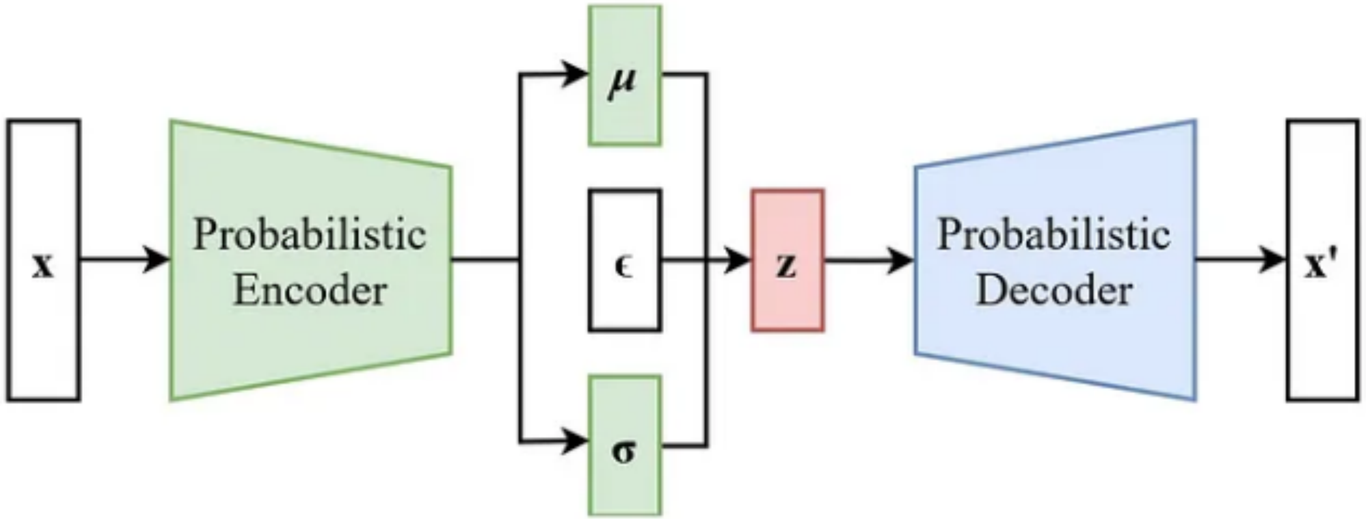

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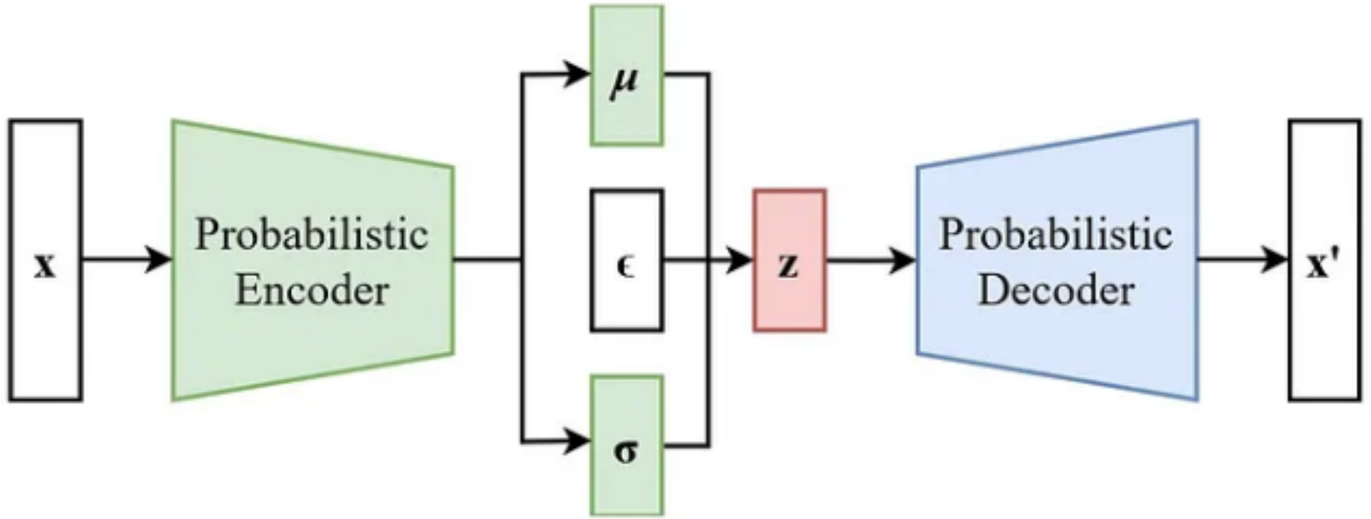
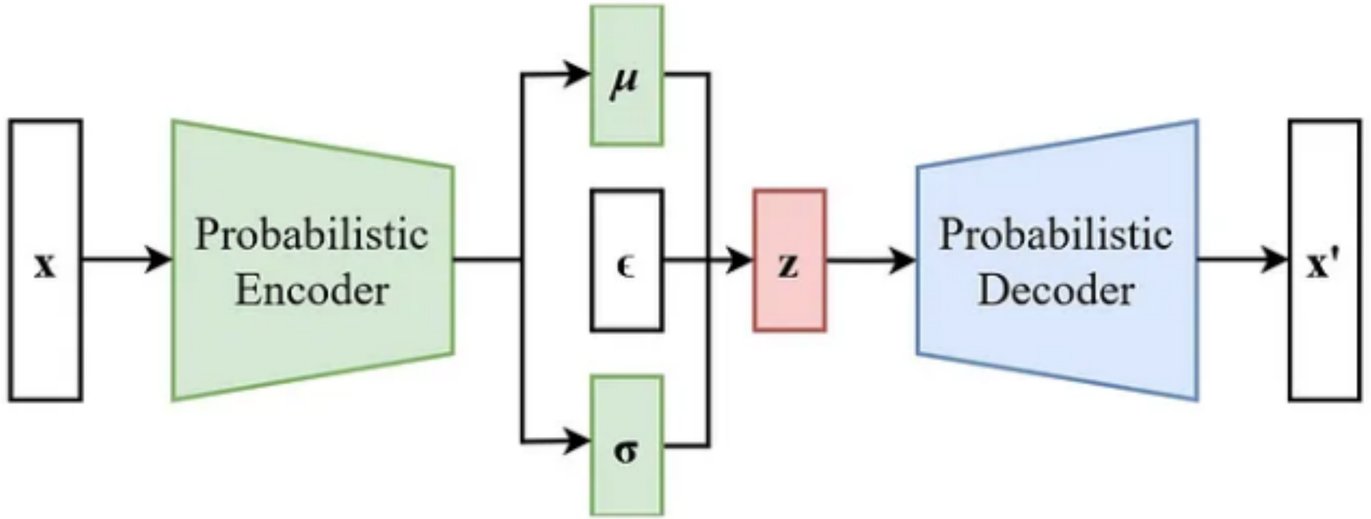


$$L(\theta_1, \theta_2) = \|x - f_{\theta_2}(z)\|_2^2 + \beta \cdot D_{KL}(q_{\theta_1}(z \mid x) \parallel \mathcal{N}(0,I))$$

Finally...



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Recon. Loss

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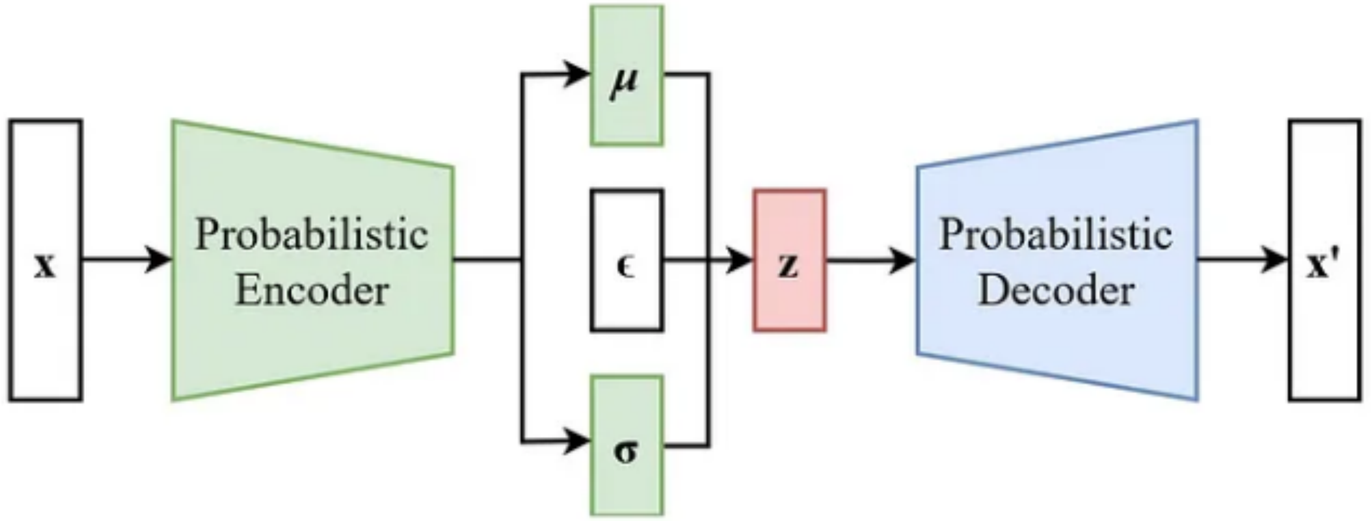

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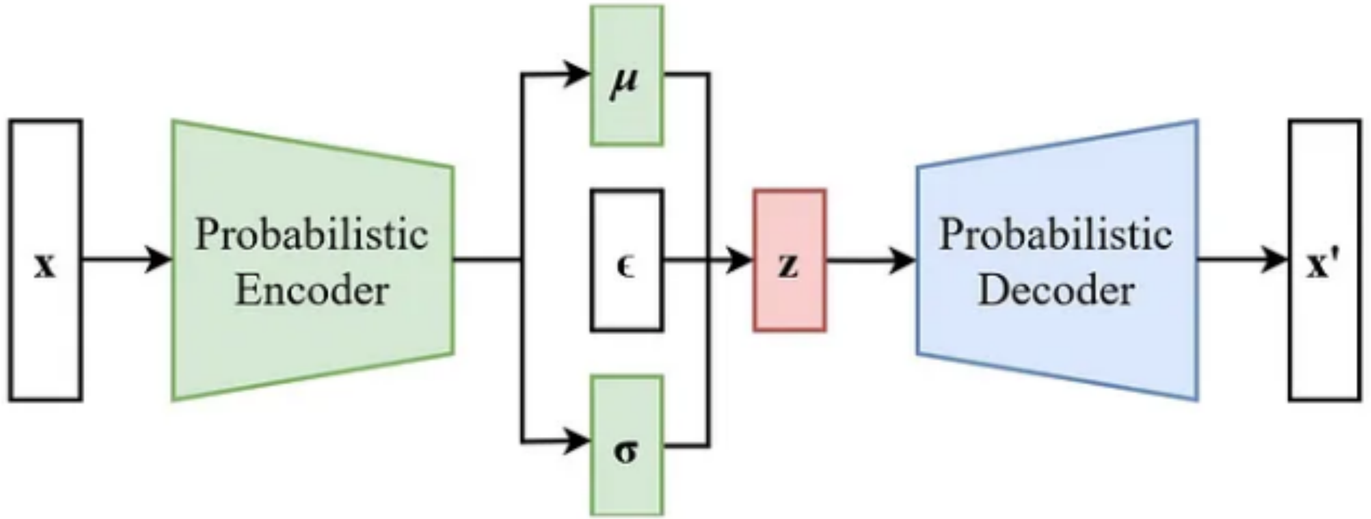
Recon. Loss

KL Penalty

Finally...



Regularization Strength

$$L(\theta_1, \theta_2) = \|x - f_{\theta_2}(z)\|_2^2 + \beta \cdot D_{KL}(q_{\theta_1}(z \mid x) \parallel \mathcal{N}(0,I))$$

Recon. Loss

KL Penalty

SOTA

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Stable Diffusion XL’s VAE reduces the dimension by 48 times

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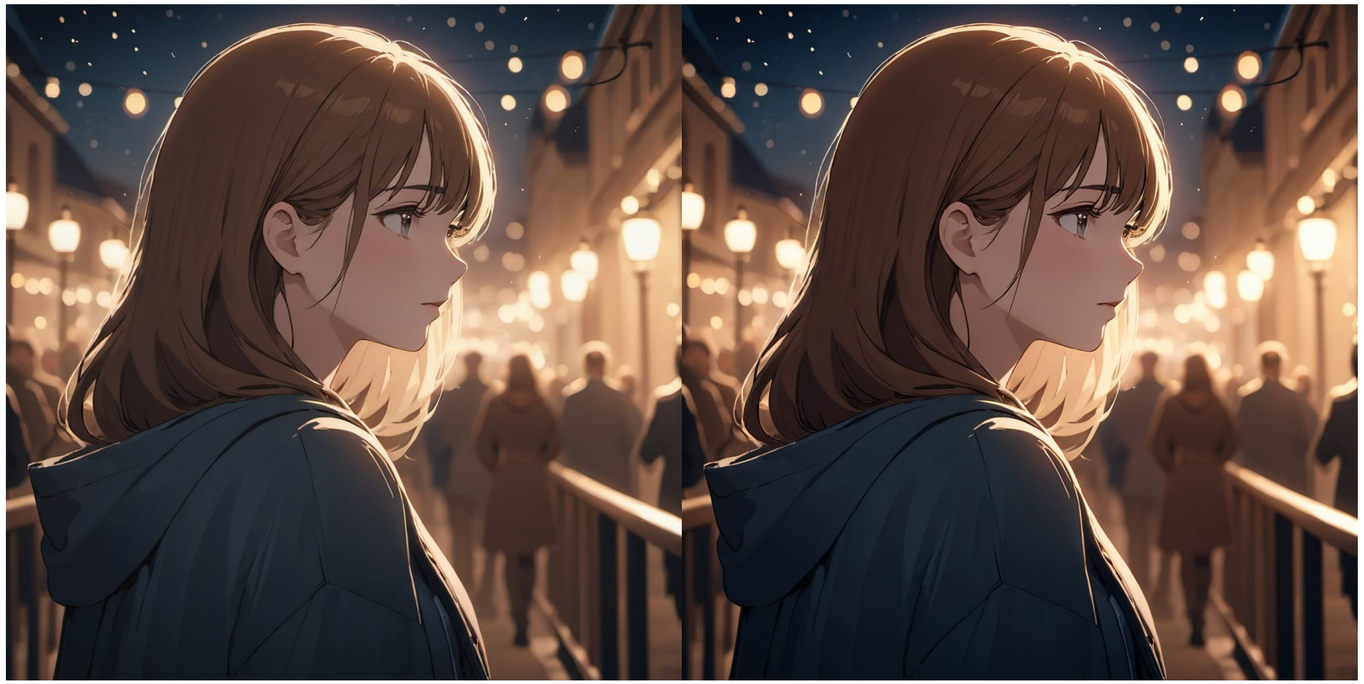
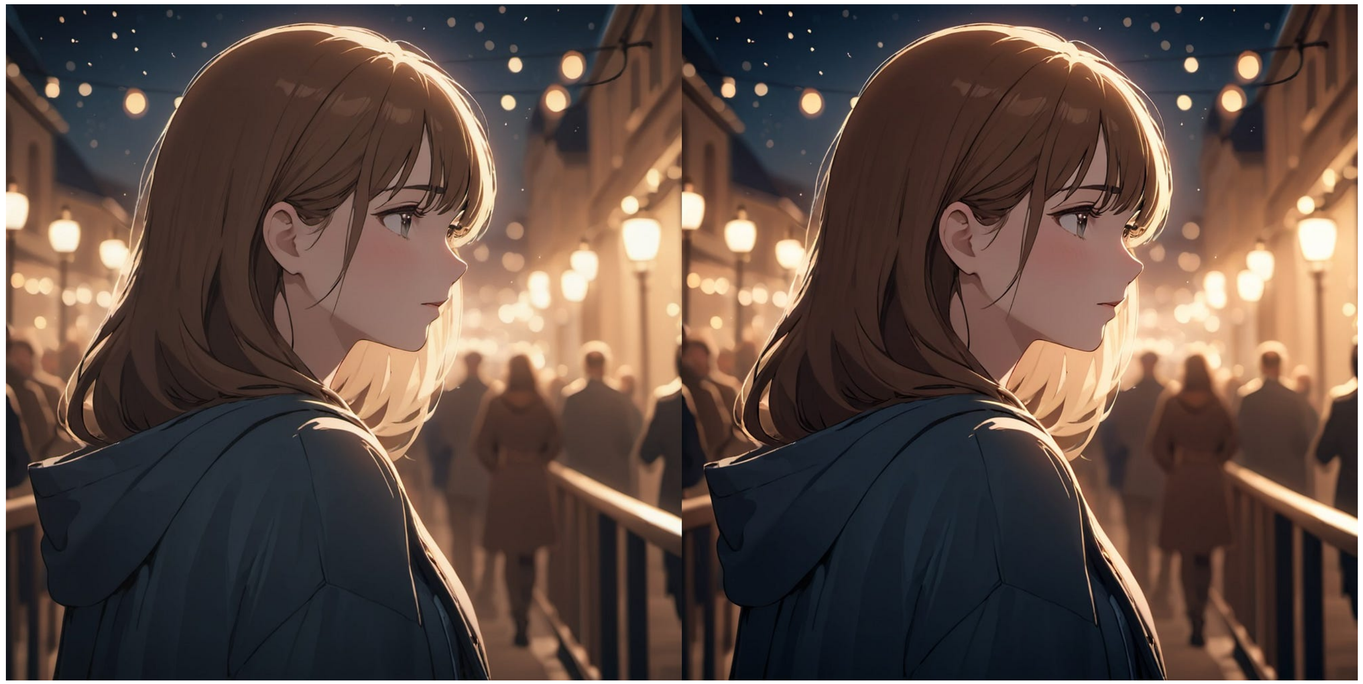


Original

Reconstructed

SOTA

Stable Diffusion XL's VAE reduces the dimension by 48 times



Original

Reconstructed

Sequence Modelling

Introduction to Machine Learning, Spring 2026

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Inputs are sequences $\{x_t\}$ and we wish to model them

Given$\{x_1, \cdots, x_{t-1}\}$ can we predict x_t?

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Examples:

- Stock prediction
- Next token prediction (language models!)
- Weather Prediction, etc

Sequence Modelling - Challenges

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Variable length inputs

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What if sequences are very long?

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Remembering “context”

Sequence Modelling - Challenges

Variable length inputs

What if sequences are very long?

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Remembering “context”

Predictions need to somehow use the entire history

Approach #1: RNNs

Key Idea

Represent the entire context as a hidden variable $\boldsymbol{h} \in \mathbb{R}^d$

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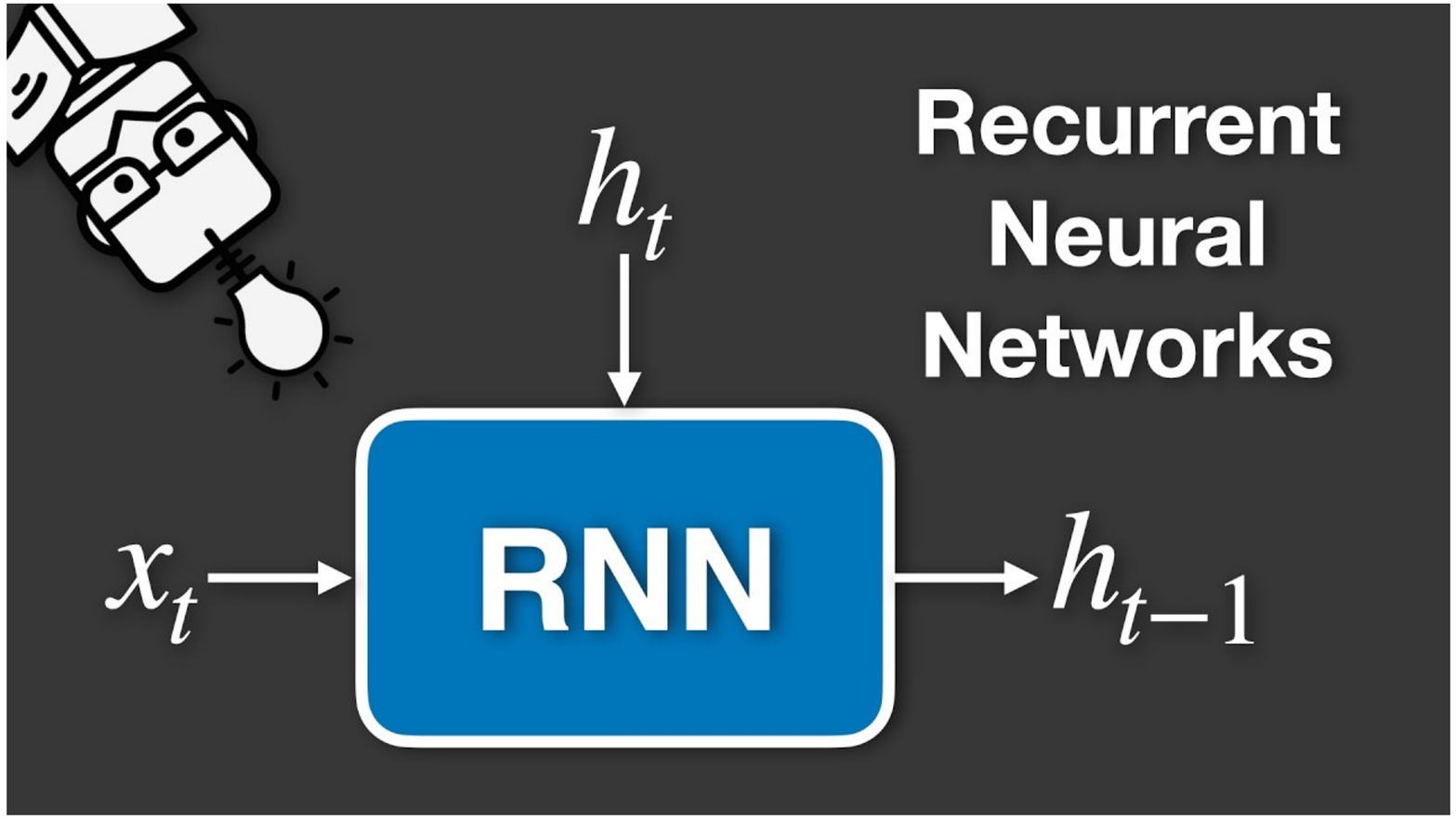
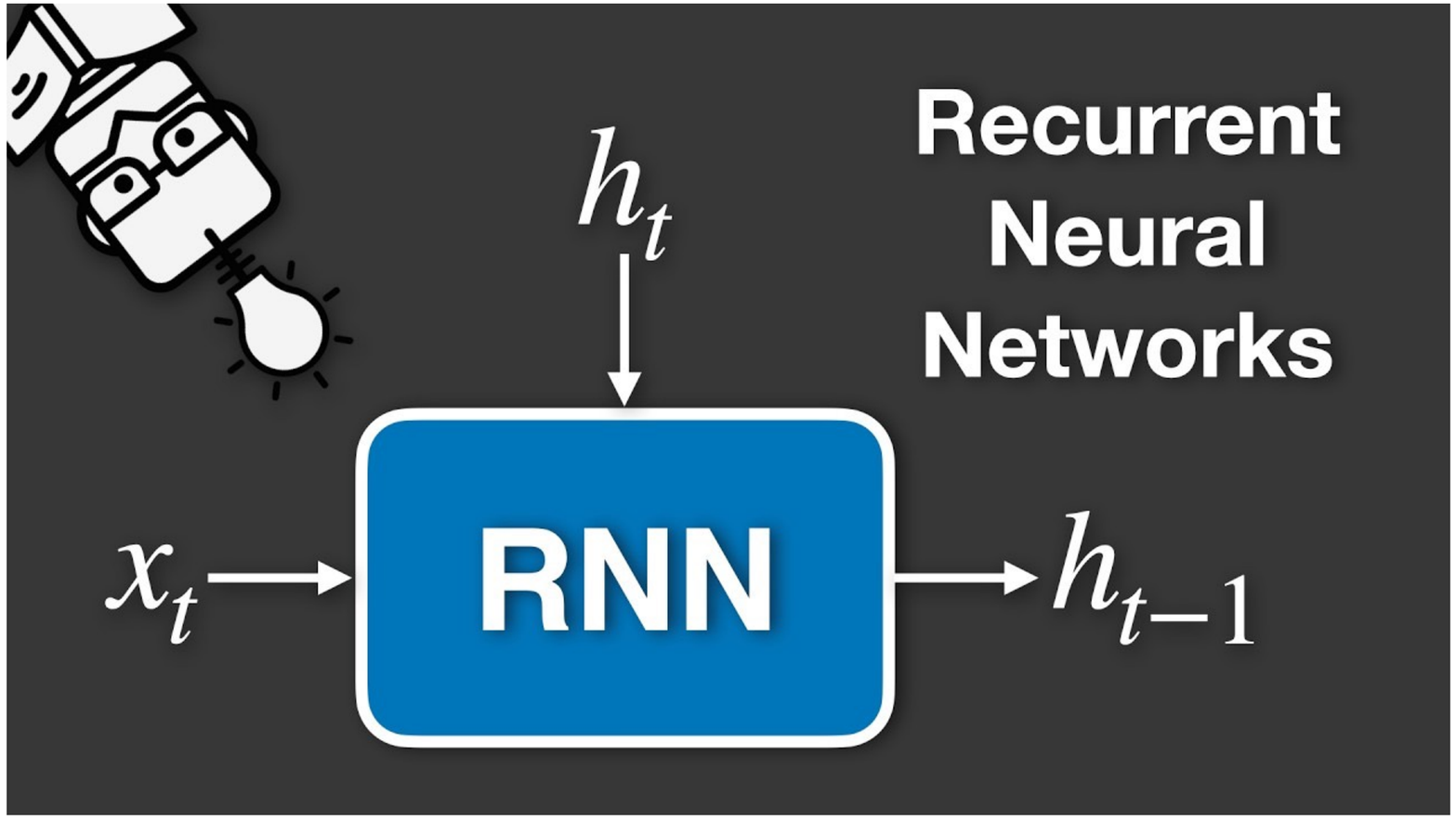
Represent the entire context as a hidden variable $\boldsymbol{h} \in \mathbb{R}^d$

Given the hidden variable, generate the next output

Recurrence?



Recurrence?



Pitfalls

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Forgets history: for a long context, the hidden state contains very little information about the starting tokens

Values and gradients either explode or vanish - unstable

Hard to parallelise - only run sequentially

Increases cost of training and inference

What now?

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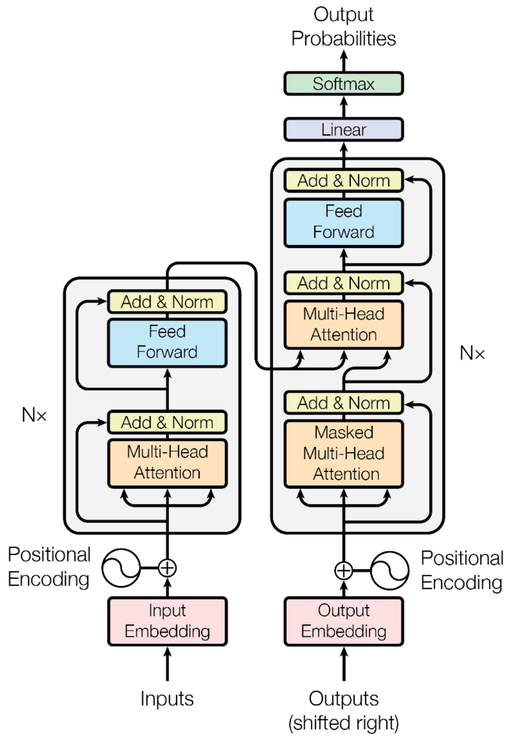
What now?

Attention Is All You Need



What now?

Attention Is All You Need